Rate Positivity for Arbitrarily Varying Channels

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I. Abstract

Shannon's work in Information theory has lead to the characterization of various *fixed channels* (such as DMCs) with independent and random noise where both the sender and the receiver are aware of the complete channel behaviour. However, further works have also led to the characterization of broadly called *varying channels* (such as compound channels, AVCs, etc.) where the communication parties remain oblivious to the complete channel behaviour. It corresponds to a more realistic scenario in wireless communication systems where the channel between the communicating parties maybe poorly characterized. Such channels can be arbitrarily controlled by an adversary who may or may not act maliciously to disrupt the communication between the legitimate parties.

The poster describes the problem of adversarial communication over such *varying channels* i.e., the Arbitrarily Varying Channel which is controlled by a malicious and omniscient adversary, *James*. The problem setup is as follows: *Alice* wants to communicate a message m to *Bob*, she encodes it as a codeword $\underline{x}$ and transmits over the AVC, the malicious adversary, *James* who is omniscient to m, $\underline{x}$, and the channel law, inputs a state sequence $\underline{s}$ in order to disrupt the communication between *Alice* and *Bob*. The AVC model considered here is assumed to be index-wise decomposable and state deterministic in nature, i.e., the channel function w is a deterministic function of the input codeword $\underline{x}$ and the state sequence $\underline{s}$. Bob, finally receives the noisy codeword $\underline{y}$ from which he must decode $\hat{m}$ such that $\hat{m} = m$. The considered problem assumes even a more general setting that includes a large family of channels by considering input type constraints at *both* Alice and James. The goal of this work is to characterize the rate positivity of the AVC under such a scenario i.e. when a positive rate of communication is possible between *Alice* and *Bob* ?
The capacity of any AVC is first demonstrated to be totally dependent on the relation between two convex sets i.e., the *confusability set* and the *completely-positive-self-coupling set* which can be characterized by single letter probability distributions. Then, an achievability and a matching converse is presented to characterize the rate positivity. *Achievability:* The zero- error capacity of an AVC is positive if the confusability set $\mathcal{K}(P_X)$ is strictly contained within the completely-positive-self-coupling set $\mathcal{CP}(P_X)$, for some distribution, P_X satisfying the input constraints. *Converse:* The zero error capacity of an AVC is zero if the completely-positive-self-coupling set $\mathcal{CP}(P_X)$ is contained within the confusability set $\mathcal{K}(P_X)$ for every distribution P_X satisfying the input constraints. The achievability uses a random code construction based on *completely-positive-self-coupling* called "cloud codes", while the converse is based on the ideas of Ramsey theory, generalization of Plotkin bound that leverages the duality result of Completely positive matrices and Copositive matrices followed by a proof based on Fourier analysis for the non-existence of a certain sequence of random variables. The above problem on state-deterministic AVCs has been already studied in [1].

Another such subclass of AVCs that have also been studied in the literature includes the non-state deterministic AVCs (eg. cascade of an Adversarial BSC(p_1) with a classic BSC(p_2)) where the channel function w is a stochastic function of input codeword $\underline{x}$ and the state sequence $\underline{s}$. Our current work focuses on coming up with a similar rate positivity characterization for the general non-state deterministic and index wise decomposable AVCs by providing an achievable code construction and a matching converse.

Remark: The Poster presentation will be based on:

[1]. X. Wang, A. J. Budkuley, A. Bogdanov and S. Jaggi, "When are large codes possible for AVCs?", in *2019 IEEE International Symposium on Information Theory (ISIT)*.
[2]. Ongoing joint work with Sidharth Jaggi (University of Bristol & CUHK), Amitalok J. Budkuley (IIT Kharagpur) and Yihan Zhang (Technion, Israel) on Rate positivity for general non-state deterministic AVCs.